

GUJARAT TECHNOLOGICAL UNIVERSITY**B.E. Sem-III (All Branches) Examination December 2009****Subject code: 130001****Subject Name: Mathematics III****Date: 15/12/2009****Time: 11.00 am – 2.00 pm****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** Do as directed **14**
- (a) Solve: $\frac{dy}{dx} + y = x$.
- (b) Evaluate the integral: $\int_0^{\infty} \exp(-x^2) dx$.
- (c) Find $L\{\sin 2t \cos 2t\}$.
- (d) State the generating function and integral representation for the Bessel function $J_n(x)$.
- (e) Prove that: $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$.
- (f) Show that: $x^3 = \frac{2}{5} P_3(x) + \frac{3}{5} P_1(x)$.
- (g) Find the Fourier transform of the function
- $$f(x) = \begin{cases} k & 0 < x < a \\ 0, & \text{otherwise} \end{cases}$$
- Q.2** (a) By using the method of Laplace transform solve the initial value problem: $y'' + 2y' + y = e^{-t}$, $y(0) = -1$ and $y'(0) = 1$. **07**
- (b) Solve the following differential equations
- (i) $2xy dx + x^2 dy = 0$ **02**
- (ii) $\frac{dy}{dx} - y = e^{2x}$ **02**
- (iii) $\frac{dy}{dx} + y = -\frac{x}{y}$ **03**

OR

- (b) (i) Using the relationship between the beta and gamma functions, simplify the expression $B(m, n) B(m + n, p) B(m + n + p, q)$. **02**
- (ii) Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of Gamma function. **02**
- (iii) State Legendre duplication formula. Hence prove that **03**
- $$B(m, m) B\left(m + \frac{1}{2}, m + \frac{1}{2}\right) = \pi m^{-1} 2^{1-4m}.$$

- Q.3 (a)** Solve the initial value problem : **05**
 $y'' + y' - 2y = 0$, $y(0) = 4$ and $y'(0) = -5$
- (b)** Given the functions e^x and e^{-x} on any interval $[a, b]$. **04**
 Are these functions linearly independent or dependent?
- (c)** Using the method of variation of parameter solve the differential **05**
 equation: $y'' + y = \sec x$.
- OR**
- Q.3 (a)** Prove that: $\frac{d}{dx}[x^{n+1} J_{n+1}(x)] = x^{n+1} J_n(x)$. **05**
- (b)** Attempt (**any three**). **09**
- (i) Express the polynomial $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomials.
- (ii) Show that $\int_{-1}^1 P_m(x) P_n(x) dx = 0$, if $m \neq n$.
- (iii) By using generating relation of Legendre polynomials, evaluate $P_n(-1)$.
- (iv) Obtain the value of $\int_{-1}^1 P_n^2(x) dx = 0$.
- Q.4 (a)** Find the Fourier series of the function $f(x) = x^2$, $-\pi < x < \pi$. **05**
- (b)** Obtain the Fourier series of periodic function **05**
 $f(x) = 2x$, $-1 < x < 2$, $p = 2L = 2$.
- (c)** Obtain the Fourier transform of the function $\exp(-ax^2)$. **04**
- OR**
- Q.4 (a)** Using the method of undetermined coefficients, solve the differential **05**
 equation: $y'' + 4y = 8x^2$.
- (b)** Using the method of series solution, solve the differential equation: **04**
 $y'' + y = 0$.
- (c)** Find the steady state oscillation of the mass-spring system governed by **05**
 the equation: $y'' + 3y' + 2y = 20 \cos 2t$.
- Q.5 (a)** Attempt (**any two**) **04**
- (i) Evaluate: $L^{-1} \left\{ \frac{1}{(s + \sqrt{2})(s - \sqrt{3})} \right\}$.
- (ii) Evaluate: $L^{-1} \left\{ \frac{3}{s^2 + 6s + 18} \right\}$.
- (iii) By using first shifting theorem, obtain the value of $L\{(t+1)^2 e^t\}$.
- (b)** Find the value of **04**
- (i) $L\{t \sin \omega t\}$
- (ii) $1 * 1$ where $*$ denote convolution product.

- (c) (i) Evaluate: $L^{-1} \left\{ \frac{s e^{-2s}}{s^2 + \pi^2} \right\}$. 06

(ii) Using convolution theorem, obtain the value of $L^{-1} \left\{ \frac{1}{s(s^2 + 4)} \right\}$.

OR

- Q.5 (a)** Find the solution $u(x, y)$ of the partial differential equation 07

$u_{xx} + v_{yy} = 0$ by method of separation of variables.

- (b)** Attempt (**any one**). 07

(i) Prove that Laplacian u in polar coordinate is

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{1}{r} \frac{\partial u}{\partial x} + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2}.$$

(ii) Find the potential inside a spherical capacitor consisting two metallic hemispheres of radius 1 ft separated by a small slit for reasons of insulation, if the upper hemisphere is kept at 110 volts and lower hemisphere is grounded.

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